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TECHNICAL NOTES

Numerical analysis of a tube with heat generation, thermal radiation, convection and axial conduction

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INTRODUCTION

THE THERMAL design of a hollow cathode of an MPD arcjet [1], tubular heat exchange elements and a calibration furnace for thermocouples requires a knowledge of the temperature distribution along the tube wall in order to achieve satisfactory performance of the device. Furthermore, it is also desirable to know how various mechanisms of heat flow in the tube will influence the peak temperature.

The present analysis considers the effects of internal radiation exchange within the tube, axial conduction within the tube wall and the variation of the convective heat transfer coefficient in the thermal entrance region. Hottel [2] has obtained a numerical solution for a short tube by division of the tube length into several isotherms and considering heat balance in each region. This resulted in a set of nonlinear algebraic equations which were solved for the wall temperature in each isothermal zone. In [3], the nonlinear integro-differential equation was written as a finite-difference analogue, then the tridiagonal system of equations was solved in order to compute temperature distribution along the tube wall.

It is the purpose of this note to develop a simple numerical technique for solving the nonlinear integro-differential equation, and also to compare results of the present analysis with the results of the finite-difference solution of Siegel and Keshock [3]. To the authors' best knowledge this method is not reported previously in the literature [4]. As will be shown, the scheme is computationally simple and convenient.

ENERGY BALANCE

The system to be analysed is illustrated in Fig. 1. A transparent gas enters the tube at a given inlet temperature $T_{g,1}$ and is heated to an average exit temperature $T_{g,2}$. A uniform heat flux q is applied to the tube wall, and its outside surface is assumed insulated. Each end of the tube is exposed to an outside environment temperature which is T_1 at the inlet and T_2 at the other end. The surface of the tube is assumed black and the heat transfer coefficient is considered to vary throughout the tube length.

The energy balance [3] can be written as

$$t_w^4(x)+H(x)\,[t_w(x)-t_g(x)] = 1+P\frac{\mathrm{d}^2t_w}{\mathrm{d}x^2} + \int_0^l t_w^4(\xi)K(|x-\xi|)\,\mathrm{d}\xi+t_1^4F(x)+t_2^4F(l-x) \quad (1)$$

with the boundary conditions

$$\frac{\mathrm{d}t_w}{\mathrm{d}x} = 0 \quad \text{at} \quad x = 0, l. \quad (2)$$

The geometrical form factors K and F are given in refs. [3, 5]. Equation (1) contains two dependent variables t_w and t_g; therefore an additional heat balance between tube wall and gas is required to relate t_w and t_g. Using heat balance, the equation for the gas temperature [3] can be obtained as

$$t_g(x) = C(x)\int_0^x \frac{C(\zeta)t_w(\zeta)\,\mathrm{d}\zeta}{\left[1+\dfrac{1}{2(\zeta+\delta)^{5/6}}\right]}+\frac{St_{g,1}}{C(x)} \quad (3)$$

where $C(x) = S\exp\left[-S\{x+3([x+\delta]^{1/6}-\delta^{1/6})\}\right]$.

Equation (3) is inserted into (1) to eliminate $t_g(x)$ which results in a single nonlinear integro-differential equation for

NOMENCLATURE

D	tube diameter
H	dimensionless convective heat transfer coefficient, $(h/q)(q/\sigma)^{1/4}$
h	convective heat transfer coefficient
k	thermal conductivity
L	length of tube
l	dimensionless length, L/D_i
P	dimensionless conduction parameter, $[k_w/(4qD_i)]\,[(D_o/D_i)^2-1]\,(q/\sigma)^{1/4}$
q	heat added per unit area at tube wall
S	Stanton number
T	temperature
t	dimensionless temperature, $(\sigma/q)^{1/4}T$
X, Z	axial length coordinates
x	dimensionless coordinate, X/D_i.

Greek symbols

δ	numerical constant
ζ	dummy integral variable
ξ	dimensionless variable, Z/D_i
σ	Stefan–Boltzmann constant.

Subscripts

g	gas
i	inner dimension of tube
o	outer dimension of tube
w	wall
1	inlet end of tube
2	exit end of tube
∞	fully developed heat transfer coefficient.

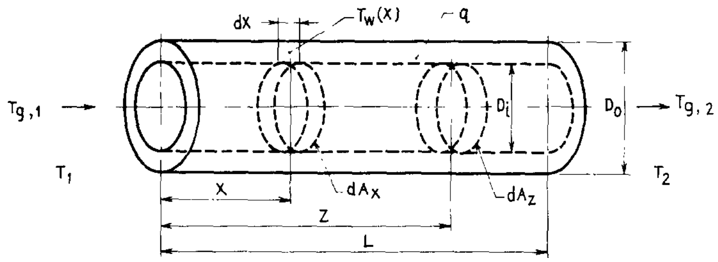

FIG. 1. Cylindrical tube geometry.

$t_w(x)$ as

$$P\frac{d^2t_w}{dx^2}-t_w^4(x)-H(x)t_w(x)$$
$$=-\int_0^x[t_w^4(\xi)K(x-\xi)+H(x)C(x)D(\xi)t_w(\xi)]\,d\xi$$
$$-\int_x^l t_w^4(\xi)K(\xi-x)\,d\xi-H(x)E(x)-A(x) \quad (4)$$

where

$$A(x)=t_1^4F(x)+t_2^4F(l-x)$$
$$D(x)=SH(x)/HC(x)$$
$$E(x)=t_{g,1}C(x)/S.$$

NUMERICAL SOLUTION AND RESULTS

The numerical integration of equation (3) is facilitated by the introduction of the transformation

$$Y_1=t_w \quad \text{and} \quad Y_2=dt_w/dx.$$

The problem is thus reduced to the solution of the following two simultaneous equations:

$$dY_1/dx=Y_2 \quad (5a)$$

and

$$P\,dY_2/dx=-1-\int_0^x[Y_1^4(\xi)K(x-\xi)$$
$$+H(x)C(x)D(\xi)Y_1(\xi)]\,d\xi$$
$$-\int_x^l Y_1^4(\xi)K(\xi-x)\,d\xi+Y_1^4(x)$$
$$-H(x)E(x)+H(x)t_w(x)-A(x) \quad (5b)$$

This set of first-order equations is solved with a standard fourth-order Runge–Kutta numerical solution technique. A satisfactory evaluation of the nonlinear integral term requires that they must be approximated to a high degree of accuracy. This is done by using Cote's formula [6]. It is now possible to solve the problem as an initial value problem starting at $x = 0$ and by initiating the calculation with trial values of $Y_1(0)$. The solution is obtained by an iterative satisfaction of the boundary condition at $x = l$. An approximate temperature profile is required to start the computation. A FORTRAN IV program has been prepared for the calculation of temperature distribution along the tube wall. The computations have been performed on CDC-CYBER-170/730 digital computer.

The numerical results of the computations are presented in Figs. 2 and 3 for the same parameters as reported in [3]. It is seen that the results of the present analysis are in fairly good agreement with results of finite-difference solution. It also reveals that the results behave fairly well over a range of parameters ($l = 10$ and 25, and $P = 5$, 10 and 25). In order to calculate computer memory requirement and CPU time, equations (1) and (2) have been solved using the finite-difference method. The solution procedure is similar to that employed by Siegel and Keshock [3]. It is important to

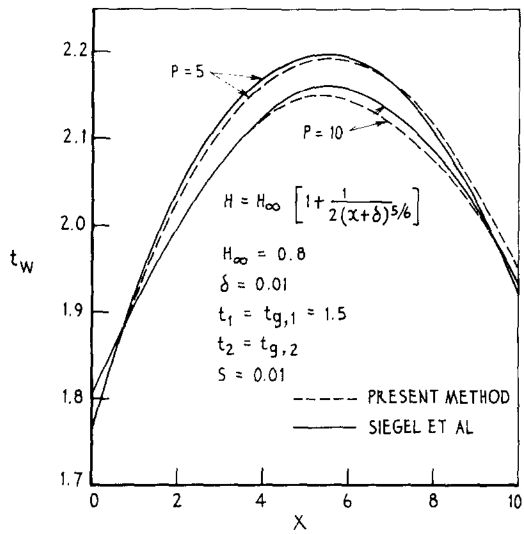

FIG. 2. Wall temperature distribution for combined conduction, convection and wall conduction for $l = 10$.

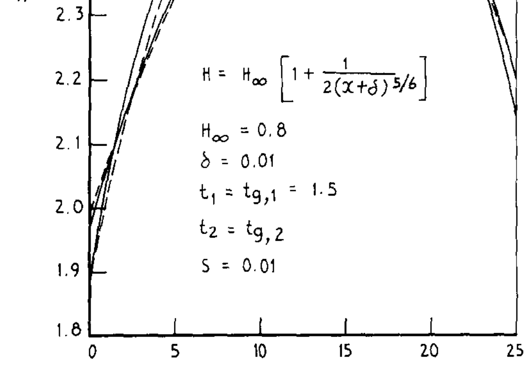

FIG. 3. Wall temperature distribution for combined conduction, convection and wall conduction for $l = 25$.

mention here that the radiation term in the LHS of (1) is linearised in the finite-difference formulation, whereas the present method does not require this linearisation. Minor improvements have however been incorporated in the program. For example, the resulting tridiagonal matrix is solved using the Thomas algorithm. For the sake of comparison, the following values are chosen in both the numerical analyses: $P = 5$, $l = 10$, step size 20 and identical initial temperature profile. It is observed that both the programs need almost the same computer memory, but the CPU time in the present method and the finite-difference method is 0.733 and 1.294 s, respectively on CYBER 170/730 computer. Therefore the present method is economical as compared to the finite-difference method. This shows that it is convenient to use the Runge–Kutta method in conjunction with an iterative scheme to solve nonlinear integro-differential equations.

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Finite-difference and improved perturbation solutions for free convection on a vertical cylinder embedded in a saturated porous medium

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1. INTRODUCTION

THE INVESTIGATION of free convection along a vertical cylinder embedded in a porous medium is of renewed interest in connection with geophysical and engineering applications. The solution of this problem within the framework of boundary-layer approximations has been obtained by Minkowycz and Cheng [1] using the local nonsimilarity method proposed by Sparrow *et al.* [2]. However, this method, which is currently very popular, has its own drawbacks as the derivatives of certain terms are discarded in order to reduce the partial differential equations to ordinary differential equations.

The object of the present paper is to give a more accurate numerical solution of the free convection boundary layer along an isothermal, thin vertical cylinder embedded in a saturated porous medium. In this respect we shall use a new implicit finite-difference scheme developed by Keller [3], and Keller and Cebeci [4] as well as the method of extended perturbation series which is similar to one devised by Aziz and Na [5] for the case of natural convection along an isothermal, thin vertical cylinder immersed in a Newtonian fluid. The specific approach is to extend the series, in terms of the transverse curvature ξ, to five terms and then apply the Shanks [6] transformation twice.

2. GOVERNING EQUATIONS

Let us consider a thin vertical cylinder of radius r_0 maintained at a uniform temperature T_w and embedded in a saturated porous medium with constant physical properties. The radial coordinate r is measured from the axis of the cylinder while the axial coordinate x is measured vertically upward such that $x = 0$ corresponds to the leading edge where the boundary-layer thickness is zero.

Based on Darcy's law and the usual Boussinesq model, the boundary-layer equations, following Minkowycz and Cheng [1], are

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \tag{1}$$

$$u = \frac{\rho_\infty g\beta K}{\mu}(T - T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{3}$$

with the boundary conditions

$$\begin{aligned} &v = 0, \quad T = T_w \quad \text{on} \quad r = r_0 \\ &u = 0, \quad T = T_\infty \quad \text{as} \quad r \to \infty. \end{aligned} \tag{4}$$

Applying the following transformations

$$\begin{aligned} \psi &= \alpha r (Ra_x)^{1/2} F(\xi, \eta), \\ ru &= \partial\psi/\partial r, \\ rv &= -\partial\psi/\partial x \\ \xi &= (2x/r_0)(Ra_x)^{-1/2}, \\ \eta &= (Ra_x)^{1/2}(r^2 - r_0^2)/(2xr_0) \\ \theta(\xi, \eta) &= (T - T_\infty)/(T_w - T_\infty), \\ Ra_x &= \rho_\infty g\beta K x (T_w - T_\infty)/(\mu\alpha) \end{aligned} \tag{5}$$

to equations (1)–(3), we find that (1) is identically satisfied and (2) and (3) reduce to

$$\theta = F' \tag{6}$$

$$(1 + \xi\eta)F''' + \xi F'' + FF''/2 = (\xi/2)[F'(\partial F'/\partial\xi) - F''(\partial F/\partial\xi)]. \tag{7}$$

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